R: Rarely I: Inconsistently C: Consistently

W S	Mathematical Modeling	W S	Mathematical Argumentation
__ __	Makes sense of problem situations, finds an entry point, & chooses and revises strategy.	__ __	Engages in problem solving collaboratively.
__ __	Flexibly uses mathematical strategies in varied mathematical contexts.	__ __	Open to revising strategies and conclusions, based on reconsideration of their own thinking and discussion with peers and teachers.
__ __	Represents problem situations and solution pathways using words, symbols, models and/or diagrams as appropriate.	__ __	Assesses whether solutions are reasonable.
__ __	Chooses & uses mathematical tools productively.	__ __	Constructs mathematical arguments using patterns and mathematical structures such as place value, behavior of operations, and geometric relationships.
__ __	Recognizes how mathematics is used in everyday and cultural practices		

W S *Number and Counting*

__ __ Count by ones forward and backward from 1 to 120 and beyond, starting at any number.
__ __ Order objects or events using ordinal numbers.
__ __ Write, compare, and order numbers to 120 and beyond.
__ __ Compose and decompose numbers to 20.
__ __ Group and count objects by tens, fives, and twos.

W S *Operations and Algebraic Thinking: Addition and Subtraction*

__ __ Develop understanding of addition in problem situations as putting together.
__ __ Develop understanding of subtraction in problem situations as take away & difference between two numbers.
__ __ Work with addition and subtraction equations and understand the meaning of the "=" sign as "same as."
__ __ Understand and use place value to compose and decompose numbers into tens and ones.
__ __ Understand and use tens and ones to add and subtract within 100.
__ __ Understand and use multiple strategies to add and subtract within 100.
__ __ Understand and use the relationship between addition and subtraction within 100.
__ __ Solve addition and subtraction word problem using various strategies.
__ __ Create addition and subtraction equations that match word problem situations within 100.

W S *Geometry: Two-dimensional and Three-dimensional shapes*

__ __ Analyze, compare, and create a variety of two- and three-dimensional figures according to their defining geometric attributes regardless of orientation or size.
__ __ Combine two- and three- dimensional shapes to create composite shapes and divide composite shapes into known shapes.
__ __ Divide circles and rectangles into two and four equal shares and describe them as halves and fourths/quarters.

W S *Measurement: Length and Time*

__ __ Use a variety of non-standard tools for measuring length.
__ __ Compare lengths using the transitive property (longer, equal to, or shorter).
__ __ Describe the connection between the size of the measurement unit and the number of units needed to measure something.
__ __ Name the days of the week and the months of the year and use a calendar to determine a day or month.
__ __ Tell and write time in hours and half hours using analog and digital clocks.

W S *Data*

__ __ Represent data using tallies, tables, picture graphs, and bar-type graphs.
__ __ Ask and answer comparison questions about data.
__ __ Conduct surveys using an accurate and clear recording system.

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SUPPORTING MATH DEVELOPMENT AT HOME: YOUNGEST LEVEL

Key principles for exploring math with your child

- *Cultivate a love of math; model curiosity*
- *Look for opportunities to mathematize; notice mathematical situations in the world around you*
- *Math isn't just about numbers: it's about building models, using logic, thinking spatially, looking for patterns*
- *Follow your child's lead and interests*
- *Be willing to not get to the answer*
- *Think out loud about your own mathematical thinking without needing a response from your child*
- *Keep it fun, conversational, and joyful!*

Mathematical practices to model and cultivate

Mathematize	Model	Explain	Strategize
Turn everyday situations into mathematical problems	Develop ways to represent problem situations	Figure out why something works	Make plans to tackle a problem

PLACES FOR MATH TALK

While commuting….	Waiting (for dinner, an appt)…	At a performance or sports event…
• Read numbers (on license plates, street signs, freeway exits). • Count objects outside the window, how many red cars? • Notice if street numbers are ascending or descending. • Try counting forwards and backwards. • Skip count forwards and backwards. If we count by 5s, will we get to 200 before we get to our destination?	• Count chairs/tables in a restaurant; estimate capacity. • Interview waiter for how many pizza boxes, napkins, etc will get used on a busy night vs. a slow night. • Play number games: I say a number, you say the one after (or the one before, 10 after, 10 before).	• Look for opportunities to read numbers and discuss their meaning. • Read scores at a sports game and figure out differences between teams. • Estimate number of people in attendance. • Read speed of pitches or track your favorite team's stats.

FAMILY ACTIVITIES TO MATHEMATIZE

Reading stories	Calendar and time	Playing games
• Make text to math connections • Read page numbers	• How many days until ___? • How many weeks?	• Discuss strategies for a good move • Keep track of points
Family routines	**Traveling**	**Other**
• How many pairs of socks does each person have in the laundry bin? • How many bags of trash do we take out every month? • How many different kinds of dishes do we have in the dishwasher?	• Use a map to figure out where you are; discuss routes; draw your own map. • Study maps, map symbols & meanings.	• Jobs for hire: 2 cents for every weed. • Measure out amounts for recipes. • Estimate cost of groceries. • Figure out budget for feeding pet. • Cut coupons; calculate savings. • Clean out junk drawers, sort and count contents.

RESOURCES FOR PROBLEM SOLVING

Games			Tools	Websites
Blokus Chess Chutes & Ladders Dominoes Farkle Golf Illusion Mancala Memory	Monopoly Origami Parcheesi Pente Rat a Tat Cat Sleeping Queens Set	Sorry Swish Sushi Go Tangrams War Yahtzee Zeus on the Loose	Clocks Counters (e.g., beans) Geoblocks; building blocks Maps Measuring tape/rulers Number line Playing cards	https://mathforlove.com/lessons/games/ https://talkingmathwithkids.com/ https://mathpickle.com/ https://denisegaskins.com/math-games/ https://www.mathanywhere.org/idea-gallery/ https://www.gamesforyoungminds.com/

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R: Rarely I: Inconsistently C: Consistently

W	S	Mathematical Modeling
__	__	Makes sense of problem situations, finds an entry point, & chooses and revises strategy.
__	__	Flexibly uses mathematical strategies in varied mathematical contexts.
__	__	Represents problem situations and solution pathways using words, symbols, models and/or diagrams as appropriate.
__	__	Chooses & uses mathematical tools productively.
__	__	Recognizes how mathematics is used in everyday and cultural practices

W	S	Mathematical Argumentation
__	__	Engages in problem solving collaboratively.
__	__	Open to revising strategies and conclusions, based on reconsideration of their own thinking and discussion with peers and teachers.
__	__	Assesses whether solutions are reasonable.
__	__	Constructs mathematical arguments using patterns and mathematical structures such as place value, behavior of operations, and geometric relationships.

W	S	**Number, Operations and Algebraic Thinking**
__	__	Understand base ten structure of numbers by reading, writing, comparing, ordering, and representing numbers to 1,000 using numbers, words, and symbols.
__	__	Round whole numbers through 1,000 to the nearest ten and hundreds.
__	__	Quickly recall basic addition facts and related subtraction facts for sums through 20.
__	__	Add and subtract two-digit numbers mentally and explain the strategies used.
__	__	Accurately add and subtract multi-digit whole numbers using strategies based on place value and explain why the procedure works.
__	__	Estimate sums and differences to determine reasonableness of answers.
__	__	Solve single- and multi-step word problems involving addition and subtraction of whole numbers and verify solutions.
__	__	Interpret and represent multiplication as finding the total of equal groups; representing the concept with arrays, equal sized groups or equal jumps on the number line. Connect representation to problem context and equation.
__	__	Interpret and represent division as equal sharing, equal jumps on the number line, and equal groups of objects. Connect representation to problem context and equation.
__	__	Apply and explain strategies to compute multiplication facts to 10 X 10 and the related division facts.
__	__	Quickly recall those multiplication facts to 10 and the related division facts.
__	__	Multiply number from 11-19 by a single-digit number using strategies based on place value and distributive property.
__	__	Solve and create single- and multi-step word problems involving multiplication and division and verify solutions.

W	S	***Number, Operations and Algebraic Thinking –Fractions***
__	__	Understand fractions are built out of unit fractions (e.g., ¾ =¼+¼+¼) & size of a fractional part is relative to the whole.
__	__	Represent fractions as wholes partitioned in equal size pieces and numbers on the number line.
__	__	Compare and order fractions that have denominators of 2, 3, 4, 6, and 8.
__	__	Represent and identify equivalent fractions understanding that two fractions are equivalent if they are the same size or the same point on a number line.
__	__	Solve single- and multi-step word problems involving comparison of fractions and verify the solutions.

W	S	***Geometry: Two- and Three-Dimensional Shapes***
__	__	Solve problems involving properties of two- and three-dimensional shapes.
__	__	Identify and describe special types of quadrilaterals.
__	__	Understand, measure, and calculate perimeters and areas of quadrilaterals.
__	__	Solve single- and multi-step word problems involving perimeters of quadrilaterals and verify the solutions.

W	S	***Measurement: Money, Linear measurement, Temperature, Time***
__	__	Develop an understanding of the U.S. monetary system by identifying each standard U.S. coin and its value.
__	__	Solve problems involving coins and bills.
__	__	Develop facility in linear measurement using standard units of measure.
__	__	Accurately use and explore a variety of measurement tools & understand when different measurement tools are required.
__	__	Estimate, measure, and compare weight, mass, and liquid volume using appropriate-sized U.S. and metric units.
__	__	Use analog clocks to tell time to the minute.

W	S	***Data: Categorical and Numerical Data***
__	__	Collect, organize, and represent categorical and numerical data with appropriate diagrams such as bar graphs, pictographs, and line plots.
__	__	Interpret and analyze bar graphs, pictographs, and line plots.

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SUPPORTING MATH DEVELOPMENT AT HOME: MID LEVEL

Key principles for exploring math with your child

- *Cultivate a love of math; model curiosity*
- *Look for opportunities to mathematize; notice mathematical situations in the world around you*
- *Math isn't just about numbers: it's about building models, using logic, thinking spatially, looking for patterns*
- *Follow your child's lead and interests*
- *Be willing to not get to the answer*
- *Think out loud about your own mathematical thinking without needing a response from your child*
- *Keep it fun, conversational, and joyful!*

Mathematical habits to model and cultivate

Mathematize	Model	Explain	Strategize
turn everyday situations into mathematical problems	develop ways to represent problem situations	figure out why something works	make plans to tackle a problem

PLACES FOR MATH TALK

while commuting….	**waiting (for dinner, an appt) …**	**at a sports event…**
• Read numbers (on license plates, street signs, freeway exits). • Ask how many more streets to get to 54th St. if we are at 47th. • Count forwards and backwards. • Skip count forwards and backwards. If we count by 2s, will we get to 200…If we count by 100, will we get to 10,000 before we get there? • Count specific items seen out the window, and look for arrays. • Determine bus routes.	• Count chairs/tables in a restaurant; estimate capacity. • Estimate number of items, people, visitors, wait time…etc. • Play number games: I say a number, you say the one after (or the one before, 10 after, 10 before). • Use ages for games: "When you are 20, how old will I be?" • Estimate price and tip price of meal. • Budgeting for meals.	• Read scores and figure out differences between teams. • Estimate number of people in attendance. • Read speed of pitches or speed of bowling ball. • Track stats for different sports. • Record own stats; discuss pertinent stats & their meaning.

FAMILY ACTIVITIES TO MATHEMATIZE

reading stories	**calendar**	**playing games**
Make a reading plan; a certain number of pages per day; how many days to finish the book.	How many days until we go to___? How many weeks? Differences in age; how many months, days, years older/younger.	Think out loud about potential strategies. Keep track of points.
family routines	**traveling**	**other**
Giving allowance: how much will it take to save up for ___? How much will I have by the end of the month?	use a map to figure out where you are; discuss routes; draw your own map Study maps, map symbols & meanings	Notice math in everyday routines and cultural practices Measure out amounts for recipes Estimate cost of groceries Figure out budget for feeding pet

RESOURCES FOR PROBLEM SOLVING

Games			**Tools**	**Websites**
Blackjack Blokus Chess Dominoes D and D	Farkle Jaipur Mastermind Monopoly Origami	Tangrams Top Trumps War Yahtzee	Cookbooks Geoblocks; building blocks Maps Measuring tape/rulers Playing cards	https://mathforlove.com/lessons/games/ https://talkingmathwithkids.com/ https://mathpickle.com/ https://denisegaskins.com/math-games/ https://www.mathanywhere.org/idea-gallery/ https://www.gamesforyoungminds.com/

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R: Rarely I: Inconsistently C: Consistently

W S	Mathematical Modeling	W S	Mathematical Argumentation
__ __	Makes sense of problem situations, finds an entry point, chooses & revises strategies, & perseveres in problem solving.	__ __	Engages in problem solving collaboratively.
__ __	Flexibly uses mathematical strategies in varied mathematical contexts.	__ __	Open to revising strategies and conclusions, based on reconsideration of their own thinking and discussion with peers and teachers.
__ __	Represents problem situations and solution pathways using words, symbols, models and/or diagrams as appropriate.	__ __	Assesses whether solutions are reasonable.
__ __	Chooses & uses mathematical tools productively.	__ __	Constructs mathematical arguments using patterns and mathematical structures such as place value, behavior of operations, and geometric relationships.
__ __	Recognizes how mathematics is used in everyday and cultural practices		

W S *Number, Operations and Algebraic Thinking: Multiplication and Division*

__ __ Understand place value to 1,000,000, understanding a digit in one place represents 10 times as much as it represents in the place to its right and $^{1}/_{10}$ of what it represents in the place to its left.
__ __ Accurately and efficiently multiply up to four-digit numbers, using and explaining both mental and written strategies as appropriate.
__ __ Solve single- and multi-step problems in varied contexts involving multi-digit multiplication.
__ __ Accurately and efficiently divide up to four-digit numbers by one- or two-digit divisors, using and explaining both mental and written strategies as appropriate.
__ __ Solve single- and multi-step problems in varied contexts involving multi-digit division.

W S *Number and Operations, and Algebraic Thinking: Fractions and Decimals*

__ __ Compare fractions with different numerators and denominators.
__ __ Identify explain why fractions are equivalent.
__ __ Accurately add and subtract fractions, including mixed numbers.
__ __ Multiply & divide fractions, explaining what multiplication and division means in the context of fractions.
__ __ Solve single- and multi-step problems in varied contexts involving addition and subtraction of fractions.
__ __ Understand decimal notation and compare decimals by reasoning about their size.
__ __ Accurately add and subtract decimals to hundredths.
__ __ Accurately multiply and divide decimals to hundredths.
__ __ Solve single- and multi-step problems in varied contexts involving addition and subtraction of decimals.
__ __ Solve single- and multi-step problems in varied contexts involving multiplication and division of decimals.

W S *Algebraic Relationships*

__ __ Determine rules for numerical patterns and extend the patterns.
__ __ Write a rule to describe the relationship between two sets of data that are linearly related.
__ __ Write algebraic expressions that represent simple situations and evaluate the expressions.
__ __ Graph ordered pairs in the coordinate plane for two sets of data related by a linear rule & draw the line they determine.

W S *Geometry: Two- and three-dimensional Shapes*

__ __ Classify quadrilaterals with explicit use of parallel and perpendicular lines and angle size.
__ __ Identify, sketch, and measure acute, right, and obtuse angles.
__ __ Identify, describe, and classify triangles by angle measure and number of congruent sides.
__ __ Determine the perimeters and areas of rectangles, triangles, and parallelograms.
__ __ Draw quadrilaterals and triangles from given information about sides and angles.
__ __ Solve single- and multi-step problem situations about the perimeters and areas of quadrilaterals and triangles.

W S *Measurement and Data: Line Plots and Volume*

__ __ Construct and interpret varied data representations including line plots.
__ __ Understand concepts of volume, including understanding of how it is measured using cubic units.
__ __ Determine volume of rectangular prisms and composite solids using measurement, and using multiplication and addition-based strategies.

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SUPPORTING MATH DEVELOPMENT AT HOME: OLDEST LEVEL

Key principles for exploring math with your child

- *Cultivate a love of math; model curiosity*
- *Look for opportunities to mathematize; notice mathematical situations in the world around you*
- *Math isn't just about numbers: it's about building models, using logic, thinking spatially, looking for patterns*
- *Follow your child's lead and interests*
- *Be willing to not get to the answer*
- *Think out loud about your own mathematical thinking without needing a response from your child*
- *Keep it fun, conversational, and joyful!*

Mathematical practices to model and cultivate

Mathematize	Model	Explain	Strategize
turn everyday situations into mathematical problems	develop ways to represent problem situations	figure out why something works	make plans to tackle a problem

PLACES FOR MATH TALK

While commuting…	**Waiting (for dinner, an appt)…**	**At a sports event…**
• Estimate time to destination and figure out actual elapsed time • Read bus and train schedules, and determine time before arrival. • Use a map to navigate • Look for decimal notations (gas stations; hiking trails; tools and discuss meanings)	• Play number games: I say a number, you say half of it, a fourth of it; triple it • Give clues to figure out a number someone is thinking of: between 100 & 130; a multiple of 4 but not 8. • Compare prices and estimate tips. • Estimate quantities of ingredients and containers.	• Record own statistics • Track stats for different teams, compare, estimate & predict • Discuss pertinent statistics & their meaning • Discuss likelihood of events (e.g., making a free-throw) • Estimate number of participants in a crowd or section.

FAMILY ACTIVITIES TO MATHEMATIZE

reading stories	**reading newspapers or magazines**	**playing games**
Make text to math connections Estimate how long it will take to finish a chapter book;	Study weather maps Interpret representations of data Look for and discuss big numbers	Discuss strategies for a good move or to win the game or solve a puzzle
family routines	**traveling**	**other**
Best buy; compare price per unit; supplies of different ingredients for favorite meals and recipes; Track elapsed and remaining time (during games, musical recordings, screen time, etc.) Notice math in everyday life	Use a map to figure out where you are; compare routes; draw your own map Study maps, map symbols & meanings; use map scale Play logic puzzles & brainteasers	Look for opportunities to use fractions, decimals, & percents How many more do I need? How much further do I have to go? How many batches of muffins can I make with this much flour? How long will this bag of pet food last?

RESOURCES FOR PROBLEM SOLVING

Games			**Tools**	**Websites**
24 Battleship Blokus Chess D and D Farkle	Illusions Monopoly Poker Prime Climb Quoridor Quarto	Tangrams Top Trumps Wits & Wagers Yahtzee	Building tools Cookbooks Maps Measuring tape/rulers Playing cards	https://mathforlove.com/lessons/games/ https://talkingmathwithkids.com/ https://mathpickle.com/ https://denisegaskins.com/math-games/ https://www.mathanywhere.org/idea-gallery/ https://www.gamesforyoungminds.com/

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